Problems for lecture 13

February 13, 2015

1. Let a and b be any two positive real numbers. Show that we can find two positive integers m, n so that
$$a < \frac{m}{n} < b.$$
Hint: Choose n large so that
$$\frac{1}{n} < b - a$$
and use the inequality above to show that you can find a positive integer m with $na < m < nb$.

2. Let A be a set of points and A' be the set of limit points of A. Find an example (without a proof) of A for each case below:
   (a) $A \subset A'$ but $A \neq A'$
   (b) $A = A'$
   (c) $A' \subset A$ but $A' \neq A$
   (d) $A' \cap A = \emptyset$ (there is no point that belongs to both A' and A)
   (e) An infinite set of points A with exactly one limit point.