Problems for lecture 14

February 19, 2015

In the problems below, you can assume that in the normal real topology, a set K is compact if and only if it is closed and bounded.

1. Decide whether the statements below are true or false. If it is true, provide a short proof . If it is false, provide a counter example.

   (a) If K is compact and F is closed then $K \cap F$ is compact.

   (b) The intersection of any collection of compact sets is compact. Hint: you can use the fact that the intersection of any collection of closed set is closed.

   (c) The union of any collection of compact sets is compact.

   (d) A finite set is always compact. Hint: is it closed and bounded?

2. Determine if each of the sets below is compact. Do explain why.

   (a) $\mathbb{Q}$

   (b) $\mathbb{Q} \cap [1, 2]$

   (c) $\mathbb{Z} \cap [0, 20]$

   (d) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots\right\}$

   (e) $\left\{1, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \cdots\right\}$