Problems for lecture 15

February 20, 2015

1. Prove that in the normal real topology, if a set K is closed and bounded then it is compact.
2. Recall that if A is bounded then $\sup A$ and $\inf A$ exist but they may not belong to A. Show that if K is a compact set then $\sup K$ and $\inf K$ exist and belong to K. Hint: using the definition of $\sup K$, can you construct a sequence of points in K which converges to $\sup K$ (for each n, look at $\sup K - \frac{1}{n}$ and argue that there is a number called $x_n \in K$ so that $\sup K - \frac{1}{n} < x_n \leq \sup K$). Be clear in your arguments.
3. Let A be a compact set and c be any real number. We define a set K to be

$$K = \{ca | a \in A\}$$

(i.e., we multiply all numbers in A by the factor c). Show that K is also compact. Hint: you can either use the definition of a compact set or use the Heine-Borel Theorem.