Problems for lecture 16

February 23, 2015

1. Prove that we cannot have a continuous function on $\mathbb{R}$ where $f([0,1]) = \mathbb{Q} \cap [0,1]$. Hint: If f were continuous, it would map a compact set to a compact set.

2. Recall that for a set A, $\max A$ and $\min A$ may not exist. If A is bounded then $\sup A$ and $\inf A$ exist but they may not belong to A. If A is compact then $\sup A$ and $\inf A$ exist and they belong to A.

   (a) Show that if K is a compact set and f is continuous on K then we can find two numbers $a, b \in K$ so that $f(a) \leq f(x) \leq f(b)$ for all $x \in K$. Hint: try to argue that $\max f(K)$ and $\min f(K)$ exist.

   (b) Is the conclusion in (a) still true if K is only a bounded set? If it is not, find a counter example. Do explain clearly.