Problems for lecture 17

In this homework, we recall the two definitions:

- A function $f(x)$ is continuous on a set A if for every $c \in A$ and every $\epsilon > 0$, we can find $\delta > 0$ (depending on c and ϵ) so that if any $x \in A$ and $|x - c| < \delta$ then
$$|f(x) - f(c)| < \epsilon.$$
- A function $f(x)$ is uniformly continuous on a set A if for every $\epsilon > 0$, we can find $\delta > 0$ (depending on ϵ) so that if any $x, y \in A$ and $|x - y| < \delta$ then
$$|f(x) - f(y)| < \epsilon.$$

Use this ϵ, δ definition for the problems below. You are not allowed to use the theorem that a continuous function on the compact set is uniformly continuous.

1. Show that if a function $f(x)$ is uniformly continuous on a set A then it is continuous on A. Be clear in your arguments.
2. Show that the function
$$f(x) = \frac{1}{x^2}$$
is uniformly continuous on $[1, 2]$.
3. Let A be any bounded set on $\mathbb{R}$ (there is a number M so that $M > |a|$ for all $a \in A$). Show that the function
$$f(x) = x^3$$
is uniformly continuous on A. Note that $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.