

## Problems for Lecture 2

January 16, 2015

1. Consider the sequence defined recursively by  $x_1 = 3$  and

$$x_{n+1} = 4 - \frac{1}{x_n}.$$

Prove by induction that  $3 \leq x_n \leq 2 + \sqrt{3}$  for all  $n$ .

2. Prove by induction that for every  $n \in \mathbb{N}$ , we have

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}.$$

3. Let  $a_n$  and  $b_n$  be two bounded sequences. Prove that the sequence  $a_n + b_n$  is also bounded. Hint: it is helpful to write down the definition of a bounded sequence and then try to use the triangle inequality.