

Problems for lecture 25

March 23, 2015

1. Find a function $f(x)$ such that $f_n \rightarrow f$ where

(a) $f_n : (0, \infty) \rightarrow \mathbb{R}$ with

$$f_n(x) = \frac{nx}{1 + nx^2}.$$

(b) $f_n : \mathbb{R} \rightarrow \mathbb{R}$ with

$$f_n(x) = \begin{cases} 1 & \text{if } |x| \geq 1/n \\ n|x| & \text{if } |x| < 1/n \end{cases}.$$

2. We consider the closed interval $[0, 1]$. Find an example of a sequence of continuous functions $f_n(x)$ on this interval such that $f_n(x)$ converges pointwise to a function $f(x)$ which is defined on $[0, 1]$ but not bounded on this interval. Hint: Think about $1/x$ and construct similar to Problem 1b.