Problems for lecture 27

March 27, 2015

1. Consider the sequence of functions

$$f_n(x) = x^{1+\frac{1}{2n-1}}$$

on $[-1, 1]$. Show that $f_n(x)$ converges uniformly to $f(x) = |x|$. Hint: show that $f_n(x)$ is an even function (i.e. $f(x) = f(-x)$) and we can focus on the case $x \geq 0$. To bound the function $|x^{1+\frac{1}{2n-1}} - x|$ by M_n, try to find the critical point and say that this function is biggest at this critical point. You may assume that

$$\lim \left(1 - \frac{1}{2n}\right)^{2n} = e^{-1}.$$

2. Recall that a function $f(x)$ is <u>uniformly continuous</u> on a set A if for every $\epsilon > 0$ we can find $\delta > 0$ so that if $|x - y| < \delta$, $x, y \in A$, then $|f(x) - f(y)| < \epsilon$. Show that if $f_n \rightrightarrows f$ and each $f_n(x)$ is uniformly continuous on A then $f(x)$ is uniformly continuous on A.