Problems for lecture 29

April 1, 2015

1. Given a sequence of differentiable function $f_n(x)$ on $[a, b]$. A common question is whether the derivative of a limit is the limit of derivatives (i.e., will we have $(\lim_{n\to\infty} f_n(x))' = \lim_{n\to\infty} f_n'(x)$?) where the limit refers to the pointwise convergence. This is certainly not true in general. The theorem in our class says this is true as soon as the convergence of $f_n'(x)$ is uniform and $f_n \to f$. Consider the sequence of functions on $[0, 1]$:

$$f_n(x) = \frac{x^n}{n}.$$

   (a) Show that $f_n(x)$ converges uniformly to a function $f(x)$ and find the formula for this function.

   (b) Part (a) implies that $\lim f_n(x) = f(x)$ (pointwise convergence). Show that $(\lim f_n(x))'$ is not the same function as $\lim f_n'(x)$ by computing these two functions. Do you see why we cannot apply the theorem above?

2. Consider the function

$$f_n(x) = \frac{nx + x^2}{2n}$$

on $[-5, 5]$. Without finding $\lim_{n\to\infty} f_n(x)$, use the theorem mentioned in Problem 1 to find $(\lim_{n\to\infty} f_n(x))'$. Do check the conditions of this theorem.