Problems for Lecture 3

January 21, 2015

1. Let c be a fixed real number. Prove that if $\lim a_n = a$ then

$$\lim_{n \to \infty} c.a_n = c.a.$$

2. Suppose that a_n is a positive sequence and $\lim a_n = a > 0$. Prove that

$$\lim_{n \to \infty} \sqrt{a_n} = \sqrt{a}.$$

   Hint: The square root conjugate formula says $|\sqrt{a_n} - \sqrt{a}||\sqrt{a_n} + a| = |a_n - a|$. Thus

$$|\sqrt{a_n} - \sqrt{a}| = \frac{|a_n - a|}{\sqrt{a_n} + \sqrt{a}} \leq \frac{|a_n - a|}{\sqrt{a}}.$$

3. (Squeeze Theorem) Show that if $a_n \leq b_n \leq c_n$ for all $n \in \mathbb{N}$, and if $\lim a_n = \lim c_n = l$, then $\lim b_n = l$. Hint: You can write $|b_n - l| < \epsilon$ as $-\epsilon < b_n - l < \epsilon$. Try to do the same for $|a_n - l|$ and $|c_n - l|$.