Problems for lecture 34

April 15, 2015

1. Recall that if a power series is convergent at a point x_0 then it is convergent absolutely at any point on $(-|x_0|, |x_0|)$. Also if the power series converges absolutely at x_0 then it converges uniformly on $[-|x_0|, |x_0|]$. Suppose we have a series $\sum a_n x^n$ which converges at every points on $(-R, R)$.

   (a) Show that the power series converges absolutely on $(-R, R)$. Hint: pick any fixed point $x \in (-R, R)$ and argue that the series is absolutely convergent there.

   (b) Show that the series converges uniformly on $[-R + \epsilon, R - \epsilon]$.

   (c) Show that the series converges uniformly on any compact subset K of $(-R, R)$. Hint: If K is compact then $\sup K$ and $\inf K$ exist and belong to K.

2. Recall the ratio test that the series of numbers $\sum b_n$ is convergent absolutely if

$$\lim \left| \frac{b_{n+1}}{b_n} \right| < 1.$$

   (a) Use the Ratio Test to show that the series $\sum n x^{n-1}$ converges absolutely on $(-1, 1)$.

   (b) Show that on any compact subset of $(-1, 1)$, we have

$$\sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$$

   where the convergence is uniform. Hint: we want to say

$$\sum_{n=1}^{\infty} n x^{n-1} = \left(\sum_{n=0}^{\infty} x^n \right)'$$

   and there are conditions for that.