Problems for lecture 34

April 17, 2015

1. Recall that a sequence of functions $s_m(x) \rightrightarrows s(x)$ on a set A if for every $\epsilon > 0$, we can find N so that if $m \geq N$ then
$$|s_m(x) - s(x)| < \epsilon \text{ for all } x \in A.$$
Use this definition to show that if $s_m(x) \rightrightarrows s(x)$ on A and $s_m(x) \rightrightarrows s(x)$ on another set B then $s_m(x) \rightrightarrows s(x)$ on $A \cup B$. Note that this is not true for the union of infinitely many sets.

2. Recall the Abel's Theorem which says if a power series $\sum a_n x^n$ is convergent at $R > 0$ then the series is convergent uniformly on $[0, R]$. Similarly, if a power series $\sum a_n x^n$ is convergent at $-R < 0$ then the series is convergent uniformly on $[-R, 0]$.
Show that if a power series converges pointwise on a set A then it converges uniformly on any compact subset K of A.
Hint: if a set K is compact then $\sup K$ and $\inf K$ exist and belong to K. Try to use Abel's theorem together with Problem 1. Note that this problem is more general than 1(c) in the previous homework set. Do not use this problem to prove 1(c) in the previous homework because the problem in the previous homework does not require the powerful Abel's Theorem.