Problems for Lecture 4

January 26, 2015

In this homework set $\inf A$ is the most lower bound of a set A. Rigorously, this means two facts

- $\inf A$ is a lower bound of A
- if c is a lower bound of A then $c \leq \inf A$.

Similarly we define $\sup A$ by

- $\sup A$ is an upper bound of A
- if b is an upper bound of A then $b \geq \sup A$.

Use this definition to rigorously prove the problems below (a sense is not a proof).

1. Prove that if
$$A = \left\{ \frac{1}{n} | n \in \mathbb{N} \right\} = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \right\}$$
then $\inf A = 0$.

2. Let A be a nonempty bounded set. Show that there are two sequences a_n and b_n where $a_n, b_n \in A$ for all n, so that
$$\begin{aligned} \lim_{n\to\infty} a_n &= \sup A \\ \lim_{n\to\infty} b_n &= \inf A. \end{aligned}$$
Hint: Use the idea of $\sup A - \epsilon$ that I mention in the lecture for a sequence of values of ϵ.

3. Let A be a nonempty bounded set and B be the set containing all the numbers $-a$ where $a \in A$, i.e.,
$$B = \{-a | a \in A\}.$$
Prove that $\inf A = -\sup B$.