

Problems for Lecture 7

January 30, 2015

1. A sequence a_n is a pseudo-Cauchy sequence if for any $\epsilon > 0$, there is an $N \in \mathbb{N}$ so that if $n \geq N$ then $|a_{n+1} - a_n| < \epsilon$. Show that if the sequence a_n is a Cauchy sequence, it is a pseudo-Cauchy sequence.
2. The goal of this problem is to give an example of a pseudo-Cauchy sequence which is not a Cauchy sequence. Consider the sequence

$$a_n = \sqrt{n}.$$

It is certainly not a Cauchy sequence since a Cauchy sequence would be bounded (we showed this in our lecture). Use the definition of a pseudo-Cauchy sequence above to show that this sequence is a pseudo-Cauchy sequence. Hint: the square root conjugate formula says

$$|\sqrt{n+1} - \sqrt{n}| = \frac{1}{\sqrt{n+1} + \sqrt{n}}.$$

Try to use an inequality to simplify the expression above.