Problems for Lecture 8

February 2, 2015

1. Prove Alternating Series Test which says that if $b_1 \geq b_2 \geq b_3 \geq \cdots \geq 0$ and $\lim b_k = 0$ then the series $\sum_{k=1}^{\infty}(-1)^{k-1}b_k$ converges. Hint: follow the proof for the convergence of series $\sum_{k=1}^{\infty}(-1)^{k-1}/k$ in the lecture.

2. This problem is known as the Comparison Test. Let a_k and b_k be two sequences satisfying $0 \leq a_k \leq b_k$ for all $k \in \mathbb{N}$. Show that if $\sum_{k=1}^{\infty} b_k$ converges then $\sum_{k=1}^{\infty} a_k$ converges. Hint: explain why

$$|a_{m+1} + \cdots + a_n| \leq |b_{m+1} + \cdots + b_n| .$$

3. Prove that if the series $\sum_{k=1}^{\infty} a_k$ converges then $\lim a_k = 0$. Hint: consider the sum

$$|a_{m+1} + \cdots + a_n|$$

that appears in the definition of a convergent series when $n = m + 1$. Be clear in your argument.