

# Quiz 1

February 12, 2015

1. You are given the list of numbers from a sample

1, 3, 5, 7

- (a) Find the median, the mean, and the variance of this list where

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}, \quad s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}.$$

Do show how you use the formulas.

We have  $\bar{x} = 4$  and

$$\begin{aligned} \bar{x} &= \frac{1 + 3 + 5 + 7}{4} = 4 \\ s^2 &= \frac{(1-4)^2 + (3-4)^2 + (5-4)^2 + (7-4)^2}{3} = \frac{20}{3}. \end{aligned}$$

- (b) Another formula to find  $S_{xx} = \sum_{i=1}^n (x_i - \bar{x})^2$  is

$$S_{xx} = \sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n}.$$

Show how you use this formula to find  $S_{xx}$  and  $s^2$ .

$$\begin{aligned} S_{xx} &= (1 + 9 + 25 + 49) - \frac{(1 + 3 + 5 + 7)^2}{4} = 84 - 64 = 20 \\ s^2 &= \frac{20}{3}. \end{aligned}$$

(c) Look at the formula for the mean

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}.$$

If we form a new list by doubling all the numbers in the previous list (replacing  $x_i$  by  $2x_i$ ), what will happen to the mean of the new list in terms of the mean of the previous list? Use the formula to explain your answer.

The mean of the new list is

$$\frac{\sum_{i=1}^n 2x_i}{n} = \frac{2 \sum_{i=1}^n x_i}{n} = 2\bar{x}.$$

Thus the mean of the new list is double that of the previous list.

(d) Look at the formula for the variance

$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}.$$

If we form a new list by doubling all the numbers in the previous list, what will happen to the variance of the new list? Use the formula to explain your answer.

The variance of the new list is

$$\frac{\sum_{i=1}^n (2x_i - 2\bar{x})^2}{n - 1} = 4s^2.$$

Thus the variance of the new list is four times the variance of the previous list.