Lecture 23

March 20, 2015

1. Try 5.2.30

2. Suppose X and Y are two discrete random variables where the possible values of X are $0, 1$ and the possible values of Y are $0, 1, 2$. Show that if X and Y are independent then $E(X)E(Y) = E(XY)$. Hint:

$$E(X).E(Y) = (0.p_X(0) + 1.p_X(1))\,(0.p_Y(0) + 1.p_Y(1) + 2p_Y(2))$$

and expand the product. Do you see where we need the independent condition ?

3. Recall the variance and the covariance

$$\begin{aligned} V(X) &= \sum x^2 p_X(x) - \mu_X^2 \\ \mathrm{Cov}(X,Y) &= \sum\sum xyp(x,y) - \mu_X\mu_Y. \end{aligned}$$

By comparing the two formulas, we see that when X, Y are the same then $\mathrm{Cov}(X,X) = V(X)$. Recall that

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sigma_X.\sigma_Y}.$$

(a) Show that when $X = Y$, $\mathrm{Corr}(X,X) = 1$.

(b) You are given that when a and c have the same sign (both positive or both negative) then

$$\mathrm{Corr}(aX+b, cY+d) = \mathrm{Corr}(X,Y).$$

Use this to show that if $Y = aX + b$ and $a > 0$ then $\mathrm{Corr}(X,Y) = 1$.