Lecture 24

March 27, 2015

1. Consider
$$f(x,y) = \frac{3}{2}(x^2+y^2)$$
when $0 \le x \le 1$ and $0 \le y \le 1$ and $f(x,y) = 0$ otherwise.
   (a) Show that $f(x,y)$ is a valid pdf function.
   (b) Find $f_X(x)$, and $f_Y(y)$.
   (c) Find the chance $P(0 \le Y \le 1/2)$.
   (d) Find $E(X+Y)$ and $E(XY)$.

2. Try 5.1.9
   (a) Hint: Use $1 = \int_{20}^{30}\int_{20}^{30} f(x,y)dxdy$ to find K?
   (b) Hint: Do you see that it is $\int_{20}^{26}\int_{20}^{26} f(x,y)dxdy$?
   (c) This part is optional. If you want to do this part, here is the hint: Use the complement rule and try to find the chance the difference is at least 2 (i.e. $Y \ge X+2$ or $Y \le X-2$). Can you explain why the answer is
$$1 - \int_{20}^{28}\int_{x+2}^{30} f(x,y)dydx - \int_{22}^{30}\int_{20}^{x-2} f(x,y)dydx.$$
   (d) Follow the definition of f_X, and f_Y in the lecture.
   (e) Look at the lecture for independent rv's.

3. Try 5.1.10. Note: A uniform distribution has the form $f(x,y) = K$ where K is a number.
   (a) Use $1 = \int\int f(x,y)dxdy$ to find K.
   (b) Do you see that we need $\int_{5.25}^{5.75}\int_{5.25}^{5.75} f(x,y)dxdy$?
   (c) This part is optional. It is similar to 5.1.9 part (c).

4. Try 5.1.13
   (a) Note that if X and Y are independent then $f(x,y) = f_X(x).f_Y(y)$.

(b) Do you see that we need $\int_0^1 \int_0^1 f(x,y)dxdy$. You can use the fact that they are independent to compute $\int_0^1 f_X(x)dx.\int_0^1 f_Y(y)dy$ instead.

(c) Do you see that we need $\int_0^1 \int_0^{2-x} f(x,y)dydx$ or $\int_0^1 \int_0^{2-y} f(x,y)dxdy$?

(d) Note that $P(1 \leq X+Y \leq 2) = P(X+Y \leq 2) - P(X+Y \leq 1)$. We found $P(X+Y \leq 2)$ in part (c). Try to find $P(X+Y) \leq 1$ with the same procedure.