Lecture 2

January 16, 2015

1. We have learned the formula below to find the sample variance
$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}. \tag{1}$$
We normally denote the numerator $S_{xx} = \sum_{i=1}^n (x_i - \bar{x})^2$.
   (a) Expand the square and separate the sum to show that
   $$S_{xx} = \sum_{i=1}^n x_i^2 - 2\bar{x}\sum_{i=1}^n x_i + \sum_{i=1}^n \bar{x}^2.$$
   (b) Another way to look at the sample mean
   $$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \tag{2}$$
   is $\sum_{i=1}^n x_i = n\bar{x}$. Use part (a) to show that
   $$S_{xx} = \sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i\right)^2}{n}. \tag{3}$$
   (c) Consider the list of numbers
   $$1, 3, 5, 7, 9.$$
   Use the formula in (1) directly to find s^2. Then use (3) to find S_{xx} from which divide $n-1$ to obtain s^2. Compare the two ways to find s^2 and make sure that you have the same answer.
2. Suppose we have a list of 5 numbers
$$x_1, x_2, x_3, x_4, x_5$$
with the mean $\bar{x}$ and the variance s^2. We construct a new list of 5 numbers by adding 2 to each number in the previous list
$$x_1 + 2, x_2 + 2, x_3 + 2, x_4 + 2, x_5 + 2.$$
   (a) Apply (2) to the new list to show that the mean of this new list is $\bar{x} + 2$.
   (b) Apply (1) to the new list to show that the variance of the new list is still the same as the variance of the previous list s^2.

Try problems 1.4.49, 1.4.53, 1.4.58 (compare the centers, the spreads, and the shapes).