Lecture 4

January 23, 2015

1. Try 2.2.13, 2.2.25, 2.2.26 (Hint: one way is to use the addition rule to find the probability of $A_1 \cap A_2$, $A_1 \cap A_3$, and $A_2 \cap A_3$ first and then it becomes similar to problem 2.2.25); 2.3.29, 2.3.31

2. Use the Venn's diagram to explain why for any three events A, B, C, we have

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C). \quad (1)$$

   Hint: you can label each disjoint area as a, b, c,... like I did in the lecture.

3. Use the Venn's diagram to explain why for any three events A, B, C, we have the two formulas

$$\begin{aligned} A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \\ A \cup (B \cap C) &= (A \cup B) \cap (A \cup C). \end{aligned}$$

4. One way to look at the formula in (1) is to group the right side in three big brackets

$$(P(A) + P(B) + P(C)) - (P(A \cap B) + P(A \cap C) + P(B \cap C)) + (P(A \cap B \cap C))$$

   where in the first bracket there is <u>ONE</u> event inside each probability (i.e., $P(A)$, $P(B)$, $P(C)$); in the second bracket there is an intersection of <u>TWO</u> events inside each probability (i.e., $A \cap B$, $A \cap C$, $B \cap C$); in the third bracket there is an intersection of <u>THREE</u> events inside the probability. Using this, could you guess the formula for the union of four events $P(A \cup B \cup C \cup D)$?

5. This is not a required problem. You should only try this if you learned induction in mathematics in Math 200. Explain the formulas in Problems 2 and 4 using the idea of induction.