Lecture 9

February 4, 2015

1. This exercise is about geometric series in Calc II that we need for this course. Recall the geometric series formula when $|r| < 1$:

$$\sum_{n=1}^{\infty} a.r^{n-1} = \frac{a}{1-r}$$

where you can think a is the first number in the series and r is the number inside the power. For example in the series

$$\sum_{n=2}^{\infty} 2.\left(\frac{1}{3}\right)^n,$$

the number a corresponds to the first number when $n = 2$ (be careful about the starting value of n), *i.e.*, $a = 2\left(\frac{1}{3}\right)^2 = \frac{2}{9}$ and $r = \frac{1}{3}$.

   (a) Use the geometric series to evaluate

$$\sum_{n=2}^{\infty} 2.\left(\frac{1}{3}\right)^n.$$

   (b) For any $|r| < 1$, explain why

$$\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}.$$

2. This problem gives more information about geometric series that we will need for the course. In this exercise, with $|r| < 1$, we want to evaluate

$$\sum_{n=1}^{\infty} n.r^{n-1}.$$

One way to do this is seeing that $nr^{n-1} = (r^n)'$ where the derivative is in the variable r. Thus

$$\sum_{n=1}^{\infty} n.r^{n-1} = \left(\sum_{n=1}^{\infty} r^n\right)'.$$

Use Problem 1 part (b) to show that

$$\sum_{n=1}^{\infty} n.r^{n-1} = \frac{1}{(1-r)^2}.$$