Quiz 10

April 27, 2015

You are given that the $(1-\alpha)100\%$ confidence interval for population proportion is

$$\tilde{p} \pm z_{\alpha/2} \frac{\sqrt{\hat{p}(1-\hat{p})/n + z_{\alpha/2}^2/4n^2}}{1 + z_{\alpha/2}^2/n}, \qquad \tilde{p} = \frac{\hat{p} + z_{\alpha/2}^2/2n}{1 + z_{\alpha/2}^2/n}$$

and when n is large this is approximately

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

The $(1-\alpha)100\%$ confidence interval for the population mean is given by one of the two cases

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \qquad \text{or} \qquad \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where the prediction interval is

$$\bar{x} \pm t_{\alpha/2} s \sqrt{1 + \frac{1}{n}}.$$

1. (3 points) We want to estimate the proportion of Americans who own their home. We take a sample of 100 Americans and see that 60% of Americans in the sample own a home. Find the 95% confidence interval for the population proportion. Explain the meaning of this confidence interval.
   We note that $\alpha/2 = 0.025$ and $z_{\alpha/2} = 1.96$. Thus approximately the confidence interval is

   $$\left(0.6 - 1.96\sqrt{\frac{0.6(0.4)}{100}}, 0.6 + 1.96\sqrt{\frac{0.6(0.4)}{100}}\right) = (0.504, 0.696).$$

   We are 95% confidence that between 50.4% and 69.6% of all Americans own a home.

2. We want to estimate the average time all Americans spend eating or drinking a day. Assume that the time spent among all Americans is approximately normal. We take a sample of 20 Americans and see that the average time spent in the sample is 1.22 hours and the sample standard deviation in the sample is 0.25 hour.

(a) (1 point) True or False? In general, the random variable

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

will have a normal distribution. True.

(b) (1 point) True or False? In general, the random variable

$$\frac{\bar{X} - \mu}{S/\sqrt{n}}$$

will have a normal distribution. False.

(c) (3 points) Find the interval where we are 99% confident that the average time spent among all Americans is in the interval.
We note that $t_{\alpha/2} = 2.861$ and the interval is

$$(1.22 - 2.861\frac{0.25}{\sqrt{20}}, 1.22 + 2.861\frac{0.25}{\sqrt{20}}) = (1.06, 1.38).$$

(d) (2 points) Find the interval where we are 98% confident that the time spent of a random American is in the interval.
We note that $t_{\alpha/2} = 2.539$ and the interval is

$$(1.22 - 2.539(0.25)\sqrt{1 + \frac{1}{20}}, 1.22 + 2.539(0.25)\sqrt{1 + \frac{1}{20}}) = (0.57, 1.87).$$