Quiz 11

April 27, 2015

You are given that the test statistic for population mean is

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \qquad \text{or} \qquad t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

and type II error is

$$\begin{cases} \Phi\left(z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) & \text{for upper test} \\ 1 - \Phi\left(-z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) & \text{for lower test} \\ \Phi\left(z_{\alpha/2} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) - \Phi\left(-z_{\alpha/2} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) & \text{for two-tailed test} \end{cases}.$$

1. The melting point of a certain vegetable oil is claimed to be 95. We check if the melting point is different from this value. A sample of 16 shows that the average in the sample is $\bar{x} = 94.32$. Assume that the distribution of melting point is normal with $\sigma = 1.20$.

   (a) Write down the null and alternative hypothesis for this problem.
   We have

   $$\begin{aligned} H_0 : \mu &= 95 \\ H_a : \mu &\neq 95. \end{aligned}$$

   (b) Check if we have enough evidence to conclude that the melting point is different from the claimed value 95 with $\alpha = 0.01$.
   The test statistic is

   $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{94.32 - 95}{1.20/\sqrt{16}} = -2.2667$$

   and $z_{\alpha/2} = 2.575$. Since z does not lie on the critical region, we do not have enough evidence to conclude that the melting point is different from the claimed value.

   (c) What type of error can we have with the conclusion in part (b)
   There is a chance that we fail to recognize that the melting point is different when it is actually different from 95. This is type II error.

   (d) If a level 0.01 test is used, find the chance that we fail to recognize that the melting point is different from 95 when the melting point is actually 94. Note that if z is too large (or too small) and does not appear on the table then $\Phi(z) = 1$ (or $\Phi(z) = 0$

correspondingly).
We note that $z_{\alpha/2} = 2.575$ and thus

$$\beta = \Phi\left(2.575 + \frac{95-94}{1.2/\sqrt{16}}\right) - \Phi\left(-2.575 + \frac{95-94}{1.2/\sqrt{16}}\right) = \Phi(5.908) - \Phi(0.76) = 1 - 0.7764 = 0.2236.$$