Quiz 7

March 20, 2015

1. You are given that $z = (x - \mu)/\sigma$. Suppose that the diameter of trees of a certain type is normally distributed with $\mu = 8.8$ and $\sigma = 2.8$.

   (a) If we sellect one random tree, what is the probability that its diameter is less than 10?
   We have
   $$P(X < 10) = \Phi(\frac{10 - 8.8}{2.8}) = \Phi(0.43) = 0.6664.$$

   (b) If we sellect four random independent trees, what is the probability that at least one has a diameter exceeding 10? Hint: Carefully apply the complement rule and part (a).
   Its complement is the case where none of the diameter exceeding 10 or all the diameters are less than 10. Thus the probability is $1 - (0.6664)^4 = 0.803$.

   (c) If we sellect one random tree, what is the probability that its diameter is between 5 and 10?
   We have $P(5 < X < 10)$ is
   $$\Phi\left(\frac{10 - 8.8}{2.8}\right) - \Phi\left(\frac{5 - 8.8}{2.8}\right) = \Phi(0.43) - \Phi(-1.36) = 0.6664 - 0.0869 = 0.5795.$$

   (d) What value c is such that the interval $(8.8 - c, 8.8 + c)$ includes 98% of all diameter values?
   We note that $\Phi(8.8 - c) = 0.5 - 0.98/2 = 0.01$. We find the z value from the table and see that $z = -2.33$. Thus
   $$-2.33 = \frac{(8.8 - c) - 8.8}{2.8}.$$
   This gives $c = 6.524$.