Quiz 8

April 27, 2015

1. Using a sophisticated calculator to compute the integral is not accepted. Suppose we have a continuous joint distribution with the pdf

$$f(x,y) = \begin{cases} x+y & \text{when } 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

   (a) Explain why this is a vaid pdf function. Show your work.
   We notice that $f(x,y) \ge 0$ and

$$\begin{aligned}
\int_0^1 \int_0^1 (x+y)dxdy &= \int_0^1 (\frac{x^2}{2} + yx)|_{x=0}^{x=1} dy \\
&= \int_0^1 y + \frac{1}{2} dy \\
&= \frac{y^2}{2} + \frac{1}{2} y|_{y=0}^{y=1} \\
&= \frac{1}{2} + \frac{1}{2} = 1.
\end{aligned}$$

   Thus $f(x,y)$ is a valid pdf.

   (b) Find the marginal probability functions f_X and f_Y. Are the two variables X, Y independent? Why?
   We have

$$\begin{aligned}
f_X(x) &= \int_0^1 x + y dy \\
&= xy + \frac{y^2}{2}|_{y=0}^{y=1} \\
&= x + \frac{1}{2}
\end{aligned}$$

   and

$$\begin{aligned}
f_Y(y) &= \int_0^1 x + y dx \\
&= \frac{x^2}{2} + yx|_{x=0}^{x=1} \\
&= \frac{1}{2} + y.
\end{aligned}$$

The two variables are not independent since $x + y \neq (x + \frac{1}{2})(y + \frac{1}{2})$.

(c) Find $E(XY)$. Show your work.
We have

$$\begin{aligned} E(XY) &= \int_0^1 \int_0^1 xy(x+y)dxdy \\ &= \int_0^1 \frac{x^3}{3}y + \frac{x^2}{2}y^2|_{x=0}^1 dy \\ &= \int_0^1 \frac{y}{3} + \frac{y^2}{2} dy \\ &= \frac{y^2}{6} + \frac{y^3}{6}|_0^1 = \frac{1}{3}. \end{aligned}$$