

Quiz 9

April 27, 2015

1. You are given that $E(\bar{X}) = \mu_{\bar{X}} = \mu$, $\sigma_{\bar{X}} = \sigma/\sqrt{n}$ and

$$Z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}.$$

The time taken by a randomly selected person to fill out an application has a normal distribution with the mean 10 minutes and the standard deviation 2 minutes.

- (a) If a random sample of five independent individuals fill out the form, what is the probability that the average amount of time taken is at most 11 minutes.

The probability is

$$\Phi\left(\frac{11 - 10}{\frac{2}{\sqrt{5}}}\right) = \Phi(1.12) = 0.8686.$$

- (b) If another random sample of six independent individuals fill out the form, what is the probability that the average amount of time taken is at most 11 minutes.

The probability is

$$\Phi\left(\frac{11 - 10}{\frac{2}{\sqrt{6}}}\right) = \Phi(1.22) = 0.8888.$$

- (c) If two independent random samples are taken, one sample of 5 as in part (a), another sample of 6 as in part (b), what is the probability that both averages in the two sample are at most 11 minutes.

Since the samples are independent, the probability is $0.8686(0.8888) = 0.7720$.

2. You are given that the $100(1 - \alpha)\%$ confidence interval for population mean is

$$\left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right).$$

We estimate the battery life for an iPod by taking a random sample of $n = 10$ and see that the sample mean is $\bar{x} = 9.47$ hours. Suppose the population standard deviation is $\sigma = 2.14$ hours and the battery life of a random iPod is approximately normal. Construct a 98% confidence interval for the mean of the battery life.

We have $\alpha = 1 - 0.98 = 0.02$, $\alpha/2 = 0.01$ and $z_{\alpha/2} = 2.33$. The confidence interval is

$$\left(9.47 - 2.33 \frac{2.14}{\sqrt{10}}, 9.47 + 2.33 \frac{2.14}{\sqrt{10}}\right) = (7.893, 11.047).$$